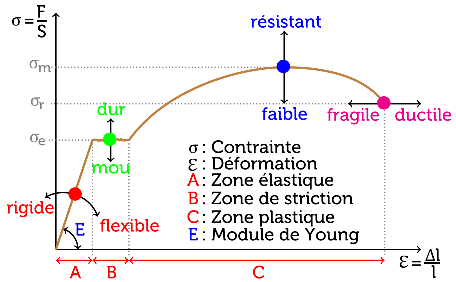
Récapitulatif de résistance des matériaux

P9-12 - Résumé

Sollicitation	$\{\mathcal{T}_{coh}\}$	$\vec{C}(M,\vec{n})$	Formules / Remarques
Extension (N > 0) **Compression (N < 0)**	$\begin{Bmatrix} N & 0 \\ 0 & 0 \\ 0 & 0 \end{Bmatrix}_G$	$\sigma\,\vec{n}$	$\boxed{N = \int_S \sigma\, dS}$ $\sigma = \frac{N}{S}$ si effort uniforme $\boxed{\sigma_x = E\mathcal{E}_x}$ $\boxed{\mathcal{E}_x = \frac{du(x)}{dx}}$ $\boxed{\mathcal{E}_y = \frac{dv(y)}{dy} = -\nu\mathcal{E}_x}$
Cisaillement	$\begin{Bmatrix} 0 & 0 \\ T_y & 0 \\ T_z & 0 \end{Bmatrix}_G$	$\tau\,\vec{t}$	$\boxed{T = \int_S \tau\, dS}$ $\tau = \frac{T}{S}$ si τ uniforme $\boxed{\tau = G\gamma}$ $\boxed{\gamma = \frac{dy}{dx}}$ $G = \frac{E}{2(1+\nu)}$
Torsion simple	$\begin{Bmatrix} 0 & \overrightarrow{m_t} \\ 0 & 0 \\ 0 & 0 \end{Bmatrix}_G$	$\tau\,\vec{z'}$	$\boxed{\theta = \frac{d\alpha}{dx}}$ $\boxed{\tau = G\theta\rho}$ $\rho = GM$ $\boxed{m_t = G\theta I_0}$ $\tau = \frac{m_t}{I_0}\rho$
Flexion plane simple	$\begin{Bmatrix} 0 & 0 \\ T_y & 0 \\ 0 & m_{f_z} \end{Bmatrix}_G$	$\sigma\,\vec{n} + \tau\vec{t}$	$\boxed{\frac{\Delta l}{l} = \underbrace{\lambda}_{>0} y}$ $\boxed{\sigma = -\frac{m_{f_z}}{I_{Gz}}y + \frac{m_{f_y}}{I_{Gy}}z + \frac{N}{S}}$ $\boxed{\tau_{xy} = \tau_{yx}}$ $\boxed{\begin{cases} EI_{Gz}y'' = m_{f_z} \\ EI_{Gy}z'' = -m_{f_y} \end{cases}}$ $\begin{cases} EI_{Gz}y'' + EI_{Gyz}z'' = m_{f_z} \\ EI_{Gy}z'' + EI_{Gyz}y'' = -m_{f_y} \end{cases}$ Flexion dérivée $\begin{cases} \text{Encastrement :} & y(0) = y'(0) = 0 \\ \text{Appuis simples :} & y(0) = y(l) = 0 \end{cases}$ Conditions aux limites
Flambement **(appuis)**	$\begin{Bmatrix} -F & 0 \\ 0 & 0 \\ 0 & -yF \end{Bmatrix}_G$	$y'' + \frac{F}{EI_{Gz}}y = 0$	$\boxed{y = B\sin\left(\frac{k\pi}{l}x\right)}$ $y = A\cos(\omega x) + B\sin(\omega x)$ $\omega^2 = \frac{F}{EI_{Gz}}$ — $\boxed{F = \frac{\pi^2 EI_{Gz}}{L^2}}$ $\begin{cases} \text{Appui simple / Appui simple : } L = l \\ \text{Appui simple / Encastrement : } L = 0{,}7l \\ \text{Encastrement / Encastrement : } L = 0{,}5l \end{cases}$
(encastré)	$\begin{Bmatrix} -F & 0 \\ 0 & 0 \\ 0 & -yF + m_B \end{Bmatrix}_G$	$y'' + \frac{F}{EI_{Gz}}y = \frac{m_B}{EI_{Gz}}$	$\boxed{y = -\frac{m_B}{F}\cos\left(\frac{2k\pi}{l}x\right)}$ $y_G = A\cos(\omega x) + B\sin(\omega x)$ $y_P = \frac{m_B}{F}$

- Limite élastique : (σ_e, τ_e)
- Limite à la rupture : (σ_r, τ_r)
- Critère de Rankine : $\sigma_{max} \leq \sigma_e$
- Critère de Guest : $\tau_{max} \leq \tau_e$
- Coef. de sécurité : $\sigma_{pe} = \frac{\sigma_e}{k}$
- Coef. de concentr° de contrainte : $\sigma_{max} = a\,\sigma_{rdm}$



Valeurs courantes :

$\nu_{\text{acier}} = 0{,}3$

$E_{\text{fontes}} = 60\text{-}160\ MPa$

$E_{\text{acier}} = 210\ MPa$

$E_{\text{cuivre}} = 120\ MPa$

$E_{\text{alu}} = 70\ MPa$

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I. Torseur

1. Définition

$$\boxed{\{\mathcal{T}\} = \begin{Bmatrix} \vec{R} \\ \overrightarrow{m_A} \end{Bmatrix}_A = \begin{Bmatrix} \vec{R} \\ \overrightarrow{m_B} = \overrightarrow{m_A} + \overrightarrow{BA} \wedge \vec{R} \end{Bmatrix}_B = \begin{Bmatrix} R_x & m_x \\ R_y & m_y \\ R_z & m_z \end{Bmatrix}_A}$$

2. Propriétés

- $\overrightarrow{m_P} \cdot \overrightarrow{PM} = \overrightarrow{m_M} \cdot \overrightarrow{PM}$
  Equiprojectivité
- $C = \vec{R} \cdot \overrightarrow{m_P} = \vec{R} \cdot \overrightarrow{m_M}$
  Invariant scalaire
- $\mathcal{P} = \overrightarrow{R_1} \cdot \overrightarrow{m_{2_P}} = \overrightarrow{R_2} \cdot \overrightarrow{m_{1_P}}$
  Comoment

Axe du glisseur : $\overrightarrow{AP} = \dfrac{\vec{R} \wedge \overrightarrow{m_A}}{R^2} + \lambda \vec{R}$

$\lambda = 0 \Leftrightarrow \vec{m} = 0$

3. Classification

- **Torseur nul** $(\boldsymbol{C} = \boldsymbol{0})$: $\{\mathcal{T}\} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}_M$ $\quad cst\ \forall\ M$
- **Torseur couple** $(\boldsymbol{C} = \boldsymbol{0})$: $\{\mathcal{T}\} = \begin{Bmatrix} 0 \\ \vec{m} \end{Bmatrix}_M$ $\quad cst\ \forall\ M$
- **Torseur glisseur** $(\boldsymbol{C} = \boldsymbol{0})$: $\{\mathcal{T}\} = \begin{Bmatrix} \vec{R} \\ \overrightarrow{m_A} \end{Bmatrix}_A$ $\quad \overrightarrow{m_A} \perp \vec{R}$
- **Torseur général** $(\boldsymbol{C} \neq \boldsymbol{0})$: $\underbrace{\{\mathcal{T}\} = \begin{Bmatrix} \vec{R} \\ \overrightarrow{m_A} \end{Bmatrix}_A}_{\overrightarrow{m_A} \pm \vec{R}} = \underbrace{\begin{Bmatrix} \vec{R} \\ \overrightarrow{m_A^1} \end{Bmatrix}_A}_{\substack{\text{Torseur glisseur} \\ \overrightarrow{m_A^1} \perp \vec{R}}} + \underbrace{\begin{Bmatrix} 0 \\ \overrightarrow{m_A^2} \end{Bmatrix}_A}_{\substack{\text{Torseur couple} \\ \overrightarrow{m_A^2} \parallel \vec{R}}}$

II. Principe fondamental de la statique (PFS)

$$\boxed{\{\mathcal{T}_{\bar{e}/e}\} = \{0\}} \Rightarrow \begin{cases} \overrightarrow{R_{\bar{e}/e}} = \vec{0} \\ \overrightarrow{m_{\bar{e}/e}} = \vec{0} \end{cases} \quad (\exists\ M \Rightarrow \forall\ F)$$

III. Torseur de cohésion

$$\boxed{\{\mathcal{T}_{coh}\}_G = \{\mathcal{T}_{E_2/E_1}\} = -\{\mathcal{T}_{\bar{E}/E_1}\} = \{\mathcal{T}_{\bar{E}/E_2}\} = \begin{Bmatrix} N & m_t \\ T_y & m_{f_y} \\ T_z & m_{f_z} \end{Bmatrix}_G}$$

N : Effort normal
T : Effort tranchant
m_t : Moment de torsion
m_f : Moment de flexion

$$\boxed{\begin{array}{ll} \dfrac{dN}{dx} = -f(x) & \dfrac{dm_t}{dx} = 0 \\ \dfrac{dT_y}{dx} = -p(x) & \dfrac{dm_{f_y}}{dx} = T_z(x) \\ \dfrac{dT_z}{dx} = -q(x) & \dfrac{dm_{f_z}}{dx} = -T_y(x) \end{array}}$$

IV. Vecteur contrainte

$$\boxed{\vec{C}(M, \vec{n}) = \frac{d\vec{f}}{dS}} = \sigma\, \vec{n} + \tau\, \vec{t} \qquad \{\mathcal{T}_{coh}\} = \int_S \{\mathrm{d}\mathcal{T}\} = \begin{Bmatrix} \displaystyle\int_S \vec{C}(M, \vec{n})\, \mathrm{d}S \\ \displaystyle\int_S \overrightarrow{GM} \wedge \vec{C}(M, \vec{n})\, \mathrm{d}S \end{Bmatrix}_G$$

V. Moments quadratiques

$\boxed{I_G = \int_S \rho^2\, dS}$ moment quadratique polaire (mm^4)

$\boxed{I_{Gy} = \int_S z^2\, dS}$ moment quadratique autour de l'axe $(G, \vec{y})$

$\boxed{I_{Gz} = \int_S y^2\, dS}$ moment quadratique autour de l'axe $(G, \vec{z})$

$\boxed{I_{Gyz} = \int_S yz\, dS}$ moment quadratique autour de l'axe $(G, \vec{z})$

$\boxed{I_G = I_{Gy} + I_{Gz}}$

1. Théorème de Huygens

$$\boxed{\begin{array}{lcrcl} I_0 & = & (y_G^2 + z_G^2)S & + & I_G \\ I_{0y} & = & z_G^2\, S & + & I_{Gy} \\ I_{0z} & = & y_G^2\, S & + & I_{Gz} \end{array}}$$

2. Valeurs importantes

Circulaire	$I_G = \dfrac{\pi D^4}{32}$	$I_{Gy} = I_{Gz} = \dfrac{I_G}{2}$	
Circulaire creuse	$\boxed{I_G = \dfrac{\pi(D^4 - d^4)}{32}}$	$I_{Gy} = I_{Gz} = \dfrac{I_G}{2}$	
Rectangulaire	$I_G = \dfrac{hb(b^2 + h^2)}{12}$	$\boxed{I_{Gy} = \dfrac{h^3 b}{12}}$	$\boxed{I_{Gz} = \dfrac{hb^3}{12}}$
Carré	$\boxed{I_G = \dfrac{a^4}{6}}$	$I_{Gy} = I_{Gz} = \dfrac{I_G}{2}$	